Instrumentation and Information Processing

Chapter 1: Signal and noise

Hiroyuki Shinoda

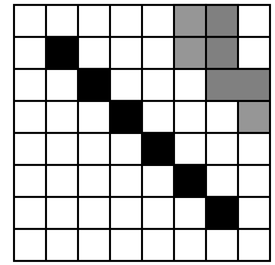
https://hapislab.org/public/hiroyuki_shinoda/keisoku_joho

hiroyuki_shinoda@k.u-tokyo.ac.jp

Contents

Providing thinking tools for design and evaluation of measurement systems

Measurement of Pattern

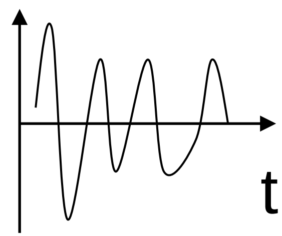


Image

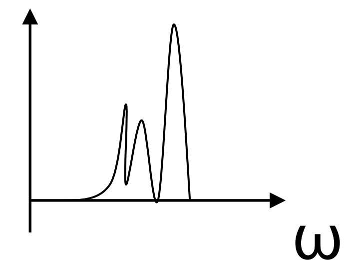
Picture -- 2D
Movie --- 3D

1） Spatial pattern

2） Temporal pattern



Time



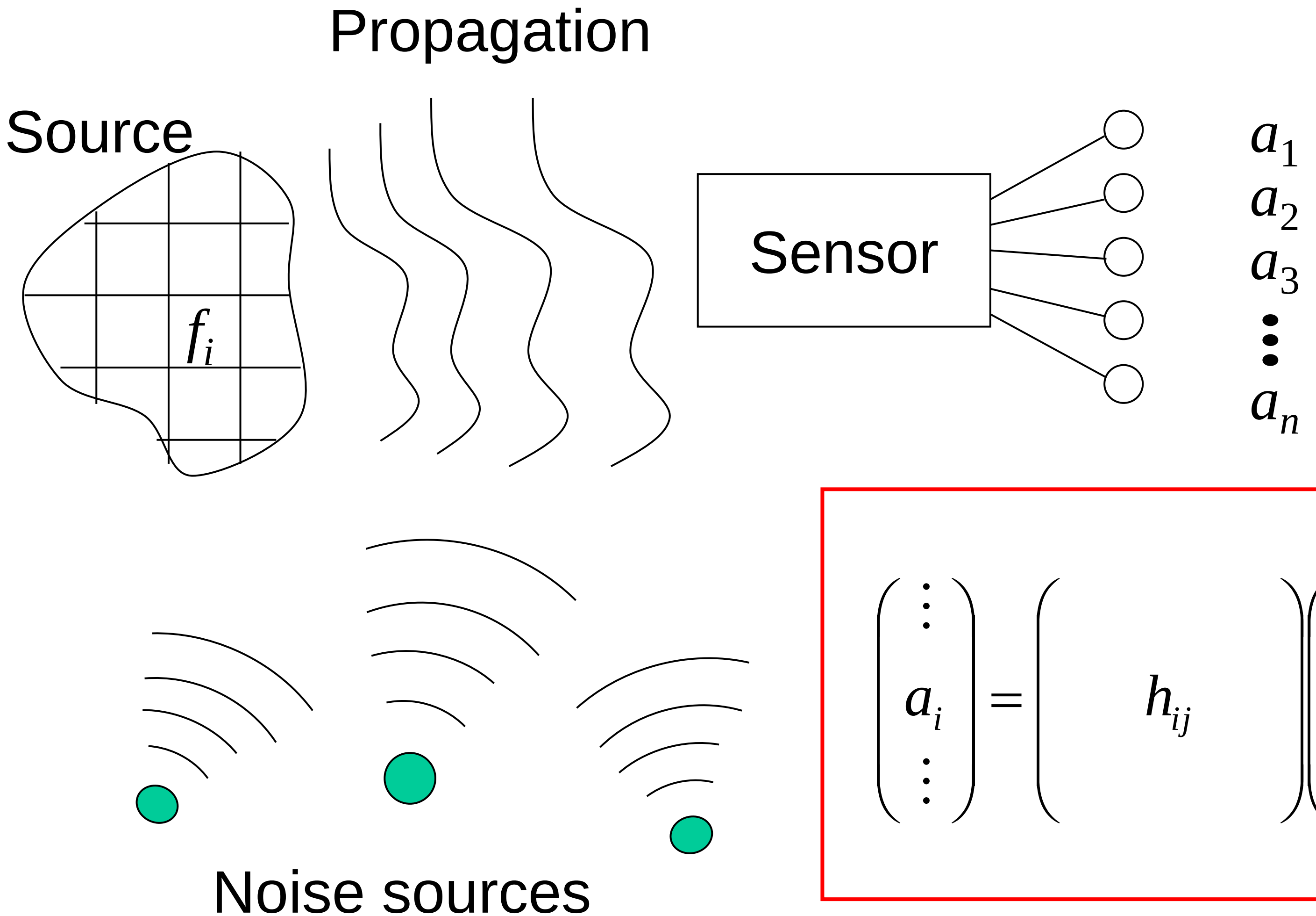
Frequency

3） Pattern of physical parameters

Optical spectrum, force vector, etc.

4） Composite of the above

Pattern measurement in linear system



Noise

$$\begin{pmatrix} \vdots \\ a_i \\ \vdots \end{pmatrix} = \begin{pmatrix} & & \\ & h_{ij} & \\ & & \end{pmatrix} \begin{pmatrix} \vdots \\ f_j \\ \vdots \end{pmatrix} + \begin{pmatrix} \vdots \\ w_i \\ \vdots \end{pmatrix}$$

Obtain $\boldsymbol{f}$ from $\boldsymbol{a}$

Classification of pattern measurement

Physical parameter

- Temperature
- Electromagnetic wave
- Electric/magnetic field
- Acoustic field
- Force/pressure/stress
- Displacement, vibration (solid, liquid, gas)
- Velocity, acceleration, angular velocity, position
- Elasticity, viscosity
- Time, frequency

Phenomenon used

- Wave reflection, transmission, absorption
- X-ray absorption
- resonance
- Nuclear magnetic resonance
- Ohm's law
- Photoelectric effect
- interference
- Moire
- Black body radiation
- Thermoelectric effect
- Hook's law
- Piezoelectric effect
- Tunneling Effect

⋮

Information needed

- Cloud distribution
- Vegetation of forest
- Location and spectrum of stars
- Surface temperature distribution
- Driving condition of car
- State of living tissue
- Thinking brain state
- Object shape
- 3D model of the environment
- Driver's fatigue
- Robot position
- Human behavior

⋮

Measurement vs. communication

[Measurement] Obtain quantities of (mainly) natural object

[Receiving signal in communication]
Obtain the signal that was intentionally sent from the sender

Phone, Wi-Fi, Bluetooth, ITS, IC tag, …

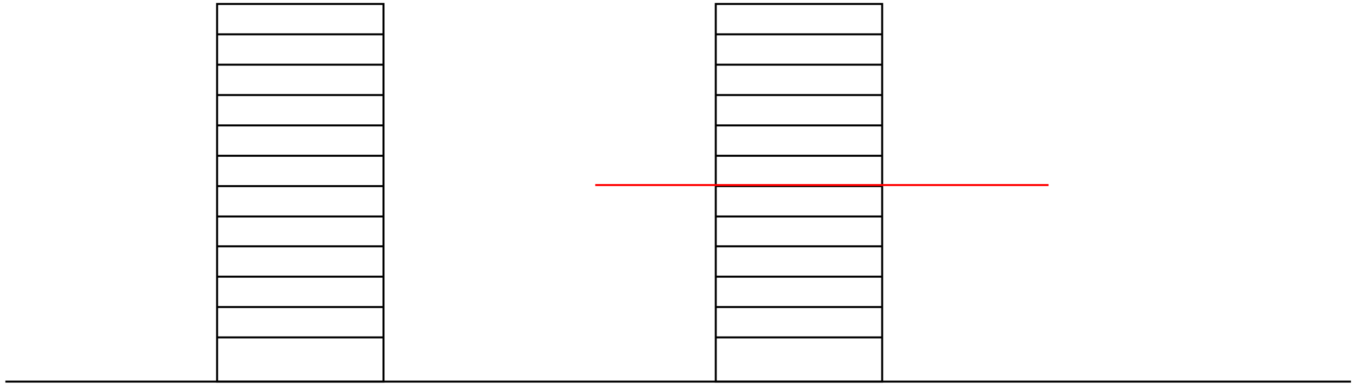
[Mixture of measurement & communication]
Sensor network, telemetry, GPS, …

Measuring signal amplitude in white noise

Goal of today

Evaluating the theoretical limit of measurement error

Quiz



A (100 coins)

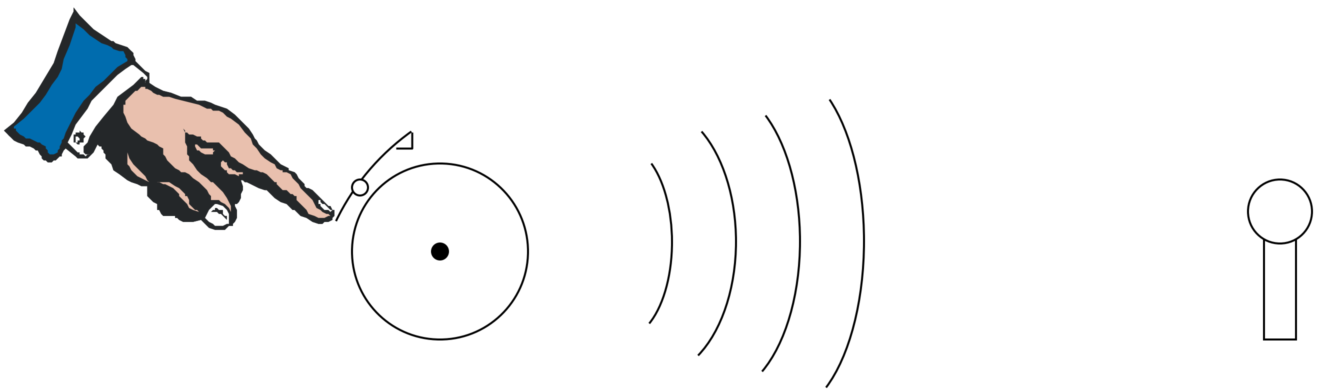
Mixture of
face-up: 30
Face-down: 70

B (100 coins)

Top half: face-up
Bottom half: face-down

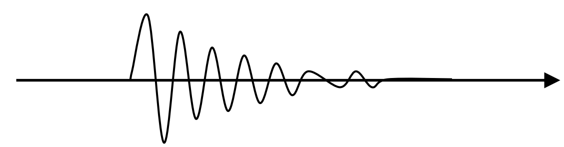
You can freely swap coins between A and B or turn them, but you cannot see the coins.
How can you make the numbers of face-up coins and face-down coins equal in both A and B?

A classical technique called “Synchronous averaging”



- $p(t) = s\ (t) + w(t)$ is observed

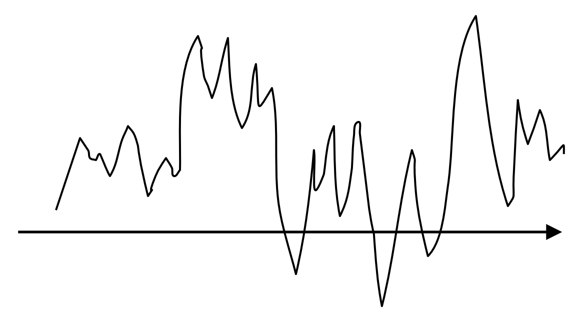
$s\ (t)$



Signal

- Finite signal length
- Same waveform every time
- Synchronous signal is obtained

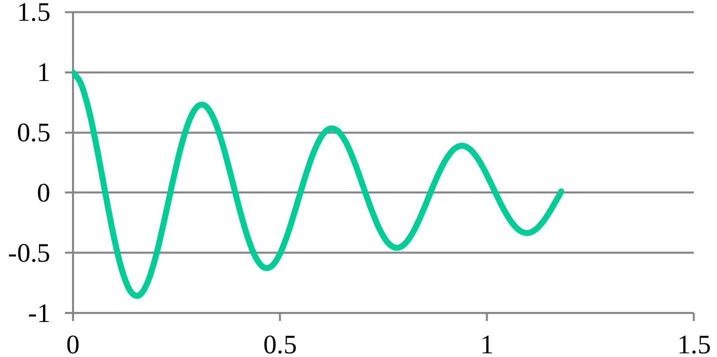
$w\ (t)$

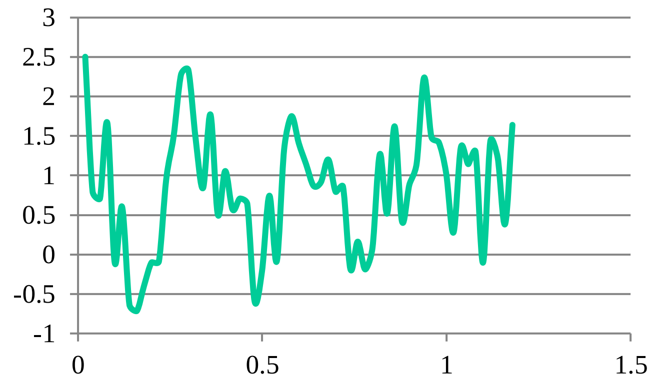


Noise

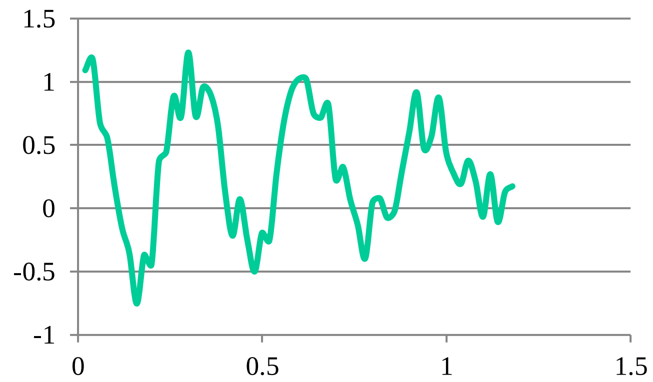
Example of synchronous averaging

Signal

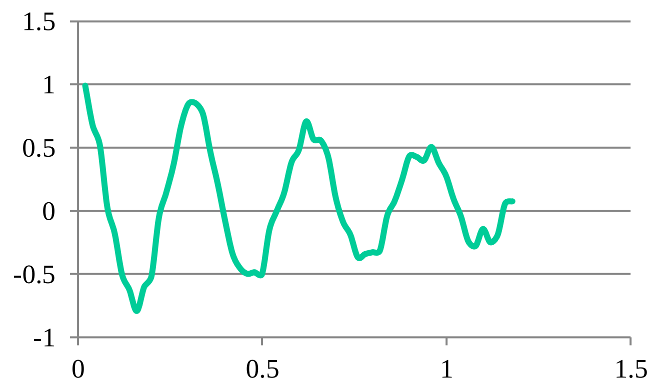




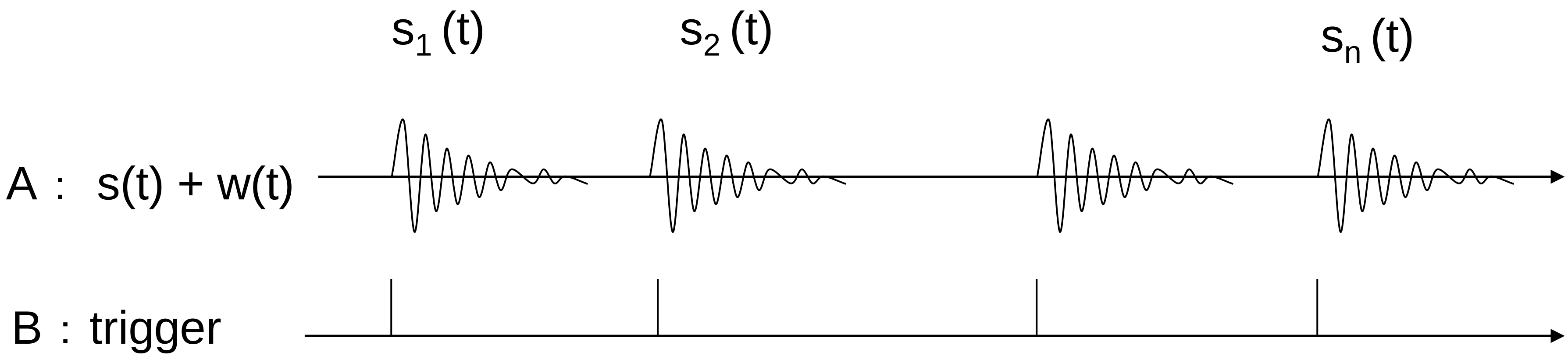
Signal + noise
（original）



10 averaging



100 averaging



$$Avg(t) = \frac{1}{N}\sum_{n=1}^{N}\{s_n(t) + w_n(t)\}$$

$$\rightarrow s(t) \quad (N \rightarrow \infty)$$

Synchronous averaging: Averaging values at each time for time-shifted signals so that the trigger signals overlap

Examples of synchronous averaging

Digital oscilloscope

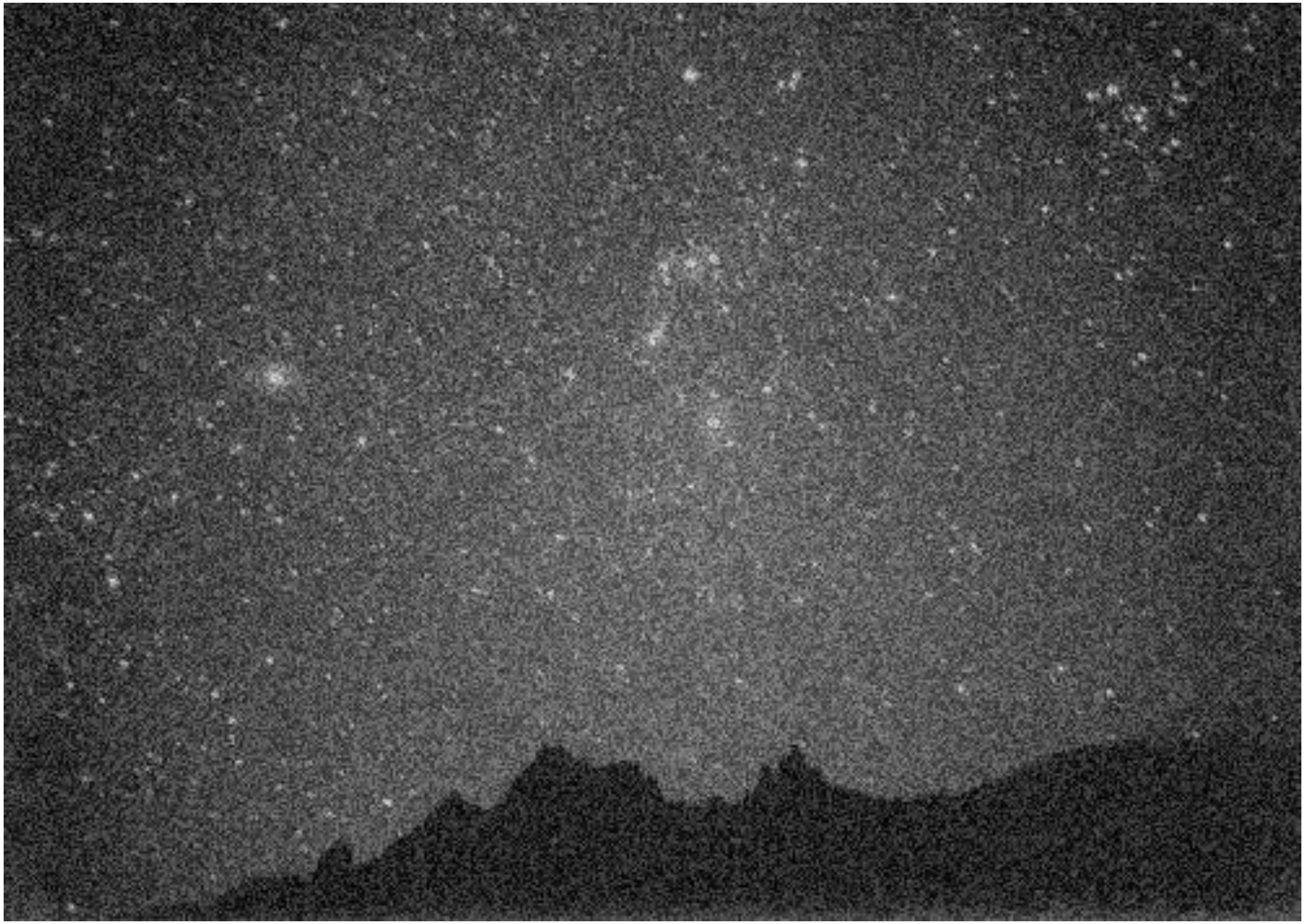
Biological signal measurement

EEG, MEG, Electrocardiography, EMG, …

Non destructive inspection

⋮

Image with noise



After averaging 100 images

Measurement model 1

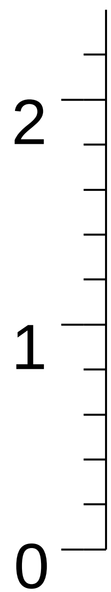
Consider a measured value p is given as

$$p = s + w$$

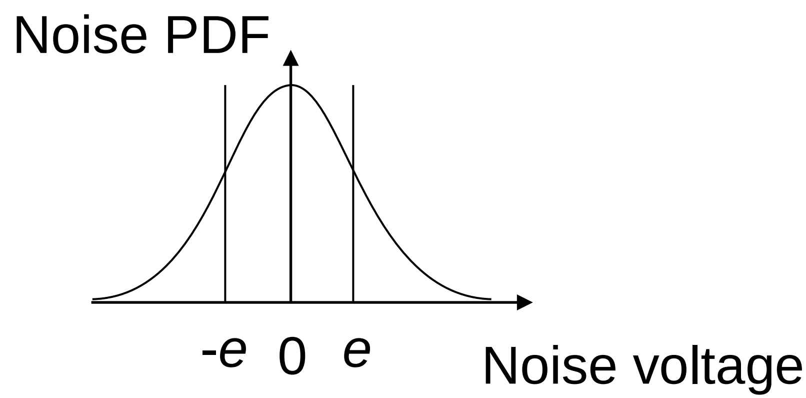
(s: the true value, w: noise).

Measured value *p*

Range: 0 ∼ V_0 [V]



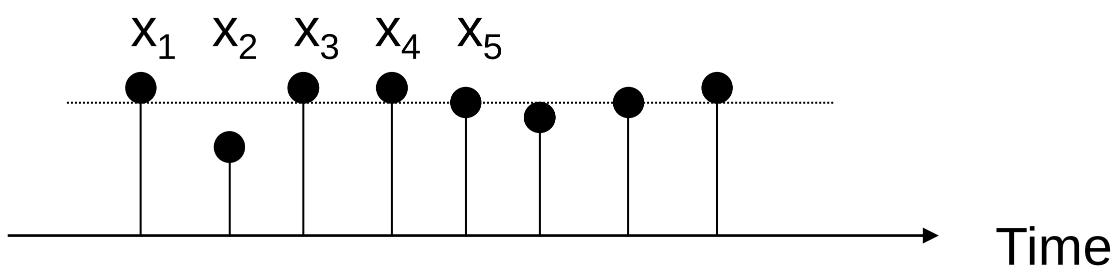
An example of noise PDF
(probability density function)



Central limit theorem for Measurement

If w is a random value, averaging multiple measured values improves the measurement error.

- prior information: True value is constant
- Measure n times
- Noise is random at every measurement



$$e_n = \frac{e_1}{\sqrt{n}}$$

e_n : Error (SD) of n-averaged data, $e_n = \sqrt{E[(p_{\mathrm{AVG}} - s)^2]}$

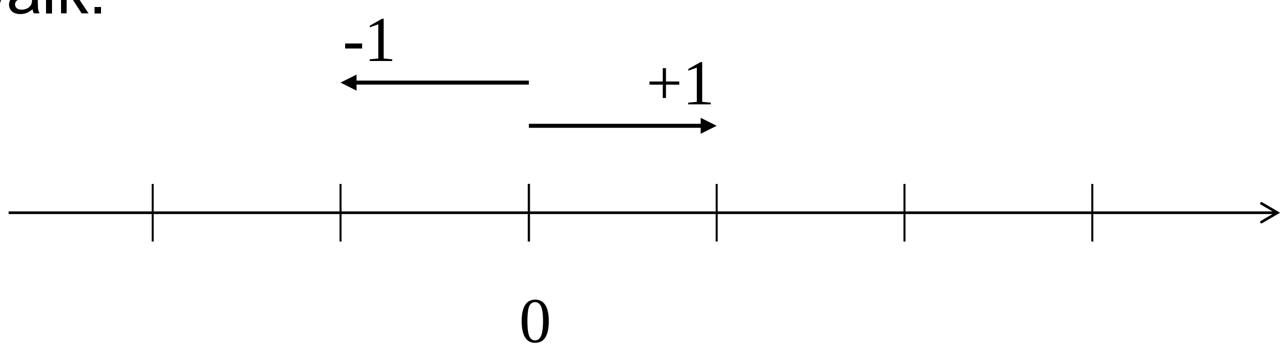
p_{AVG} : Average of n-data

Understanding $e_n = \dfrac{e_1}{\sqrt{n}}$

e_n : Error (Standard deviation) of n-averaged data

<u>A special case: Random walk</u>

In the case that $w = w_0$ or $-w_0$ (with 50% probability), it is "Random walk."



$d(n)$: Position after n trials $d(n) = a_1 + a_2 + \cdots + a_n, \quad a_i = \pm 1$

Expected value of $d(n)$: $E[d(n)] = 0$

Variance of $d(n)$:

$$V[d(n)] = E[\{d(n)\}^2] = E[a_1^2 + a_2^2 + \cdots + a_n^2] = n$$

General case

Assume w_i: random variable of variance σ^2 and average 0

The average and variance of

$$w = w_1 + w_2 + \cdots + w_n$$

are respectively written as

$$E[w] = 0$$

and

$$V[w] = E[w^2] = E[(w_1 + w_2 + \cdots + w_n)^2]$$

$$= E[w_1^2 + w_2^2 + \cdots + w_n^2] \quad \left(E\left[w_i w_j\right] = 0 \text{ for } i \neq j\right)$$

$$= n\sigma^2.$$

Therefore the standard deviation of $w \propto \sqrt{n}$.

Summary up to now

Assume the observed value is the sum of a constant true value and a random noise. Then the standard deviation of the error of the average of n-measurements decreases in proportion to $\frac{1}{\sqrt{n}}$.

Note: This does not hold unless the noise has a random value in each measurement.

Topic from now

From the case “that the true value is constant”
to the case that “the true value has a certain pattern”

Caution

The pattern ϕ in the next topic might be seen as the signal $s(t)$ shown in “Synchronous averaging,” but that is not so.

The synchronous averaging is an example of the case that the true value is constant.

Measurement model 2

Consider measured values are given as

$$\boldsymbol{p} = \boldsymbol{s} + \boldsymbol{w}$$

that is a N dimensional vector.

$\boldsymbol{p}$: N dimension vector with components of N measure values
$\boldsymbol{s}$: True value (signal)
$\boldsymbol{w}$: White noise independent from $\boldsymbol{s}$

Signal energy $$S = ||\boldsymbol{s}||^2 = \sum_{i=0}^{N-1} s_i^2$$

Noise energy $$W = ||\boldsymbol{w}||^2 = \sum_{i=0}^{N-1} w_i^2$$

Problem 1

Signal $\boldsymbol{s} = A\boldsymbol{\phi}$ is a N dimensional vector composed of N data where $\left||\boldsymbol{\phi}|\right| = 1$ and A: constant scalar.

Assume $\boldsymbol{\phi}$ is known. Then obtain the minimum value of the signal energy detectable in a white noise whose energy is W, where $\boldsymbol{w}$ is not correlated with $\boldsymbol{s}$.

Answer

It is always possible to decompose the noise $\boldsymbol{w}$ as

$$\boldsymbol{w} = a\boldsymbol{\phi} + \boldsymbol{w}'$$

where $\boldsymbol{w}'$ is a vector orthogonal to $\boldsymbol{\phi}$.

In evaluation of A from the measured N data, the above a is the inevitable error.

Since $\boldsymbol{s}$ and $\boldsymbol{w}$ are independent from each other, the expected value of a^2 is given as

$$E[a^2] = \frac{W}{N}.$$

Therefore, the signal energy S must be larger than W/N to be detected in the white noise with energy W.

Summary up to here

The expected value of noise energy allocated to a component along the signal is give as

$$E[a^2] = \frac{W}{N}$$

- N data signal is considered.
- Random noise is assumed.
- In normal distribution, the probability for

  $$|w_i| > 2.58\sqrt{W/N}$$

  is 1 %.

(Supplementary explanation)

Measured vector $\boldsymbol{p}$ can always be decomposed as

$$\boldsymbol{p} = p_0\boldsymbol{\phi} + p_1\boldsymbol{\psi}_1 + p_2\boldsymbol{\psi}_2 + \cdots + p_{N-1}\,\boldsymbol{\psi}_{N-1}$$

Parallel to the signal

Arbitrarily selected orthonormal basis (orthogonal to $\boldsymbol{\phi}$)

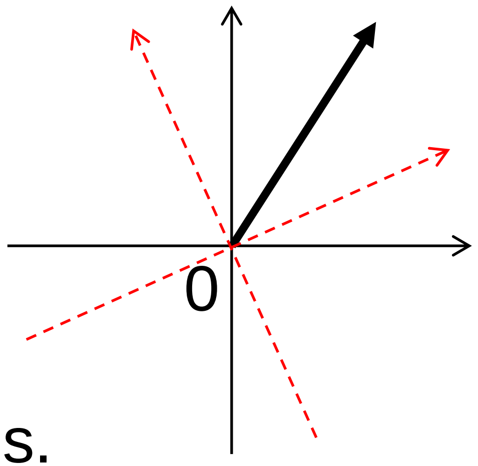
$$\boldsymbol{f} = a_1\boldsymbol{\psi}_1 + a_2\boldsymbol{\psi}_2 + \cdots + a_N\,\boldsymbol{\psi}_N$$

The energy of the signal $\boldsymbol{f}$ is given as

$$F = a_1^2 + a_2^2 + \cdots + a_N^2$$

that is independent on the choice of basis.

(Parseval's theorem)



(Supplementary explanation 2)

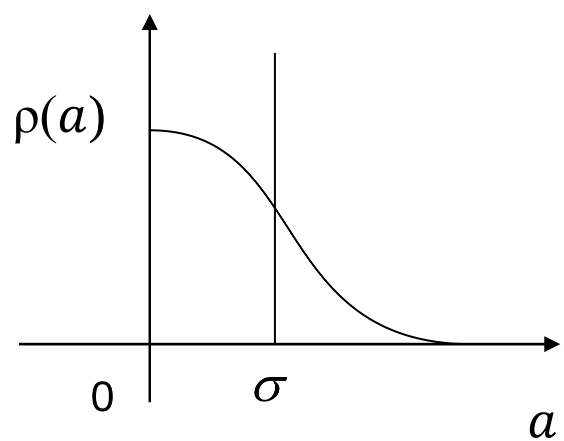
<u>$\rho(a)$: Probability density of a</u>

Parameter a of the former slide is given as

$$a = \boldsymbol{w} \cdot \boldsymbol{\phi} = \sum_{i=0}^{N-1} w_i \phi_i \,.$$

If w_i and w_j $(i \neq j)$ is independent from each other and their variance is σ^2, $\rho(a)$ converges to a Gaussian distribution with variance σ^2 as N increases. (Central limit theorem)

$$\rho(a) \rightarrow \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{a^2}{2\sigma^2}\right\}$$



∗ Note $||\boldsymbol{\phi}|| = 1$.

Derivation of the variance of a in the previous slide

$$E[a^2] = E[(w_0\phi_0 + w_1\phi_1 + \cdots + w_{N-1}\phi_{N-1})^2]$$

$$= E[(w_0\phi_0)^2 + (w_1\phi_1)^2 + \cdots + (w_{N-1}\phi_{N-1})^2]$$

$$\left(\because\ E\left[w_i w_j\right] = 0 \text{ for } i \neq j\right)$$

$$= \sigma^2(\phi_1^2 + \phi_2^2 + \cdots + \phi_n^2)$$

$$= \sigma^2$$

∗ We will change the expressions of $\boldsymbol{s}$ and $\boldsymbol{\phi}$ in the following.

Problem 1-a Signal amplitude measurement

We measure the effective value of a time-series signal under a white noise with energy W. Obtain the inevitable measurement error of the effective value.
As the prior information, the signal waveform is given as $s(n) = A\phi(n)$ with an unknown parameter A where $n = 0, 2, 3, \ldots, N-1)$

∗ In the following, N-point time-series signal $s(n)$ and $\phi(n)$ are dealt as "N dimensional vectors."

Answer 1-a

The best estimation of A is given as

$$\overline{A} = \sum_{n=0}^{N-1} p(n)\,\phi(n)$$

with the inevitable error a whose standard deviation is

$$\sigma_A = \sqrt{E[a^2]} = \sqrt{\frac{W}{N}}\,.$$

Therefore, the standard deviation of the estimated effective value (= root mean square value, RMS) is written as

$$\sigma = \frac{\sigma_A}{\sqrt{N}} = \frac{\sqrt{\mathrm{W}}}{N}$$

Question 1

Averaging $f(n)$ for $n = 1, 2, \ldots, N$ is equivalent to obtaining the vector component parallel to

$$\phi(n) = C$$

when we consider $f(n)$ and $\phi(n)$ as N dimensional vectors. Confirm this.

Question 2

Consider estimating the signal amplitude from $p(n)$.
Prove that obtaining A that minimizes

$$E = \frac{1}{N}\sum_{n=0}^{N-1}\left\{p(n) - A\phi(n)\right\}^2$$

is equivalent to conducting production of $p(n)$ to $\phi(n)$.

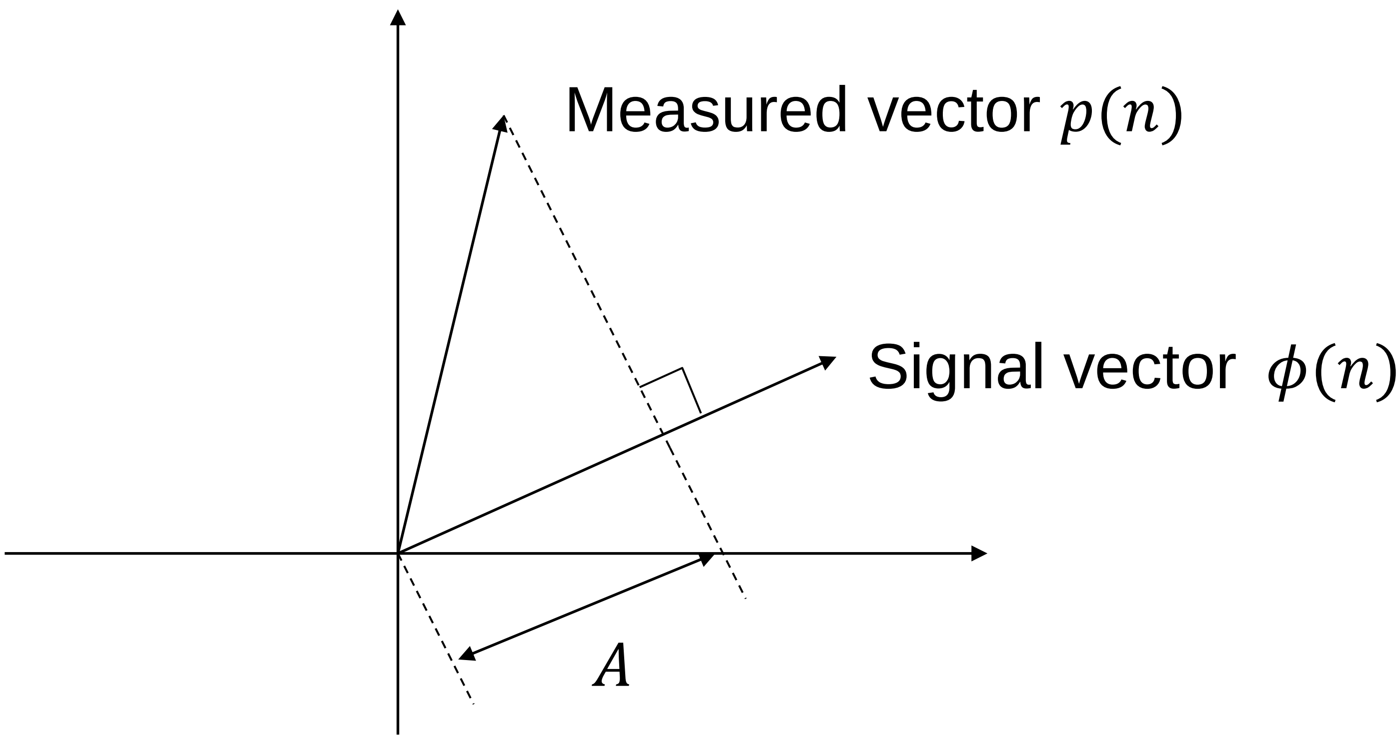
Projection

$$\sum_{n=0}^{N-1}\{p(n) - A\phi(n)\}^2$$

Obtain A to minimize the above

=

Obtain the component included in $f(n)$ parallel to $\phi(n)$.



Summary up to here

1. Noise can be decomposed into a component parallel to the signal and other orthogonal components to the signal.

2. If the signal direction (fundamental waveform) is known in the N dimensional space, the “component orthogonal to the signal” included in noise can be removed by post-observation processing.

3. The coefficient a of signal direction component $a\boldsymbol{\phi}$ contained in noise gives a measurement error. Typically, the mean and variance of a are the parameters that represent the error, which gives an accuracy limit if there is no other information to determine a.

4. When a N dimensional vector is measured and each vector component has a random value that is uncorrelated with the signal, the variance of a (expected value of a^2) is W/N where the noise energy is W.

⇨ Apply them to real problems

About the term "error"

1. It is assumed that unpredictable noise is added to the true value. “Unpredictable” does not mean that there is no information about noise. Its average and variance are known. In that case, the “measurement error” is not a single deterministic value, but expressed as the statistical parameters of the mean and the variance (standard deviation). However, if there is no misunderstanding, the standard deviation is simply expressed as "measurement error".

2. In the multidimensional measurement, it was assumed that the expected values of the noise energy of each component (degree of freedom) were equal, and the total sum of the energy was a constant.

3. The discussion in this chapter assumes that the error mean is 0, but it often nonzero in practice as offset of electronic circuit, aged deterioration, etc.) Also, in the actual measurement system, you must distinguish "Resolution", which is the smallest step of observable change from "Absolute accuracy" in which the measured values are repeatedly measured and reproduced. However, these are not distinguished here.

Problem 1

A sinusoidal wave of a known frequency f $[\mathrm{Hz}]$ was observed for T $[s]$ under a white noise with the frequency density d $[\mathrm{V}/\sqrt{\mathrm{Hz}}]$.

Obtain the inevitable estimation error in the effective value for $f \gg 1/T$.

$\boxed{\text{Noise density}}$

What does this mean?

$$2.0\,\frac{\mathrm{V}}{\sqrt{\mathrm{Hz}}}$$

Example: If the frequency band is 100 Hz, the effective value of the noise is

$$2.0 \times \sqrt{100} = 20 \ \ [\mathrm{V}]$$

Preparation

- Fourier transform $f\ (t) \rightarrow F\ (\omega)$ is a kind of "change of basis"

  Discrete Fourier transform: $f(n)\ \rightarrow F(k)$

  - Different frequency components $\cos(2\pi k_1 n)$ and $\cos(2\pi k_2 n)$ $(k_1 \neq k_2)$ are orthogonal to each other. $\cos(2\pi kn)$ and $\sin(2\pi kn)$ are also orthogonal.

- The effective value of the noise after applying a bandpass filter with the bandwidth B (to white noise $w(t)$) is proportional to $\sqrt{B}$.

  ∵ The sum of squares of the frequency components (considering a discrete signal) is proportional to the original signal energy. (It is possible to make them completely equal by appropriately selecting the coefficient of the Fourier transform.)

  The spectrum outside the passband of the bandpass filter may be arbitrary, that is, $w(t)$ does not have to be white noise.

Answer

Suppose N point data are obtained by sampling the observed signal (after low-pass filtering with the cut-off frequency of B [Hz], the Nyquist rate). Then, the sampled white-noise energy is written as

$$W = d^2 BN. \qquad (N = 2BT)$$

If the signal phase is known, the estimation error of the signal effective value (the standard deviation of the estimated value) is given as

$$\sigma = \sqrt{\frac{W}{N}\frac{1}{N}} \quad \text{(Problem 1-a)}$$

$$= d\sqrt{\frac{B}{N}} = d\sqrt{\frac{B}{2BT}} = \frac{d}{\sqrt{2T}} \quad [\mathrm{V}]$$

Answer 2

A direct solution without sampling

The Fourier transform of a signal with a signal length T has the frequency step size $\Delta f = \frac{1}{T}$.

Therefore, the effective value of the noise allocated to one frequency component is $d\sqrt{\Delta f} = d/\sqrt{T}$.

Since there are two signals with the same signal frequency (orthogonal to each other), the energy of the noise allocated to a specific phase component is half that value, and its effective value is $d/\sqrt{2T}$.

RMS measurement error when the signal phase is arbitrary

Estimated error (standard deviation) of A for a phase-fixed signal $s(n) = A\cos(\omega t)$

$$\sigma_{\mathrm{A}} = \frac{d}{\sqrt{2T}}$$

Estimated error (standard deviation) of B for a phase-fixed signal $s(n) = B\sin(\omega t)$

$$\sigma_{\mathrm{B}} = \frac{d}{\sqrt{2T}}$$

Therefore, the variance of $C = \sqrt{A^2 + B^2}$ is given as

$$E[(C - C_0)^2] = E[C^2 - 2CC_0 + C_0^2] = E[C^2] - C_0^2$$

$$= \sigma_{\mathrm{A}}^2 + \sigma_B^2$$

$$= \frac{d^2}{T} \qquad \left(\text{Standard deviation: } \frac{d}{\sqrt{T}}\right)$$

Problem 2 --- Measurement of phase

Consider measuring the phase of a sinusoidal signal at frequency f $[\mathrm{Hz}]$ for a duration T $[s]$ $(f \gg 1/T)$ under a white noise with the frequency density d $[\mathrm{V}/\sqrt{\mathrm{Hz}}]$. Express the inevitable estimation error of the phase.

The effective value of the signal is b, and you can assume the noise amplitude is much smaller than the signal amplitude.

Answer of Problem 2

Let the true value of the signal be $s(t) = \sqrt{2}b\cos(\omega t)$ and consider the possibility that this is observed as being shifted by a minute phase θ. The signal difference due to this shift is approximated as

$$\Delta s(t) = \sqrt{2}b\cos\{\omega t - \theta\} - \sqrt{2}b\cos(\omega t) \sim \sqrt{2}\theta b\sin(\omega t)$$

If the energy $(\theta b)^2 N$ (N: data number) of this difference signal $\sqrt{2}\theta b\sin(\omega t)$ is smaller than the expected value of the energy (allocated to one degree of freedom) $(d^2/(2T))$, the true waveform can be observed as the θ-shifted signal by the added noise and it is impossible to discriminate from the data whether it is due to the noise or due to a true signal shift.

If we consider the expected value of the difference signal energy is equal to the expected value of the noise energy allocated to one degree of freedom, we obtain $E[(\theta b)^2 N] = \frac{d^2}{2T}N$, thus,

$$E[\theta^2] = \frac{d^2}{2b^2T}$$

which provides the inevitable variance of the signal phase estimate.

Problem 3 --- Measurement of Pulse arrival time

The noise is white and uncorrelated with the signal, and the noise density is d. The signal is a pulse-like waveform $s(t) = r(t - t_0)$ localized near $t = t_0$ $(0 < t_0 < T)$.
The waveform $r(t)$ is known and we want to estimate the unknown parameter t_0 from the signal observed waveform during $0 < t < T$.

Obtain the estimation error bound of t_0.
$s(t)$ is a continuously differentiable function and you may use $P = \int_0^T \left(\frac{d}{dt} s(t)\right)^2 dt$ in the answer.

Answer of Problem 3

Consider the difference function for Δt-shift as

$$\Delta s(t) = s(t+\Delta t) - s(t) \sim \Delta t \frac{d}{dt} s(t) = \Delta t s'(t)$$

and the expected valued of its energy.

The energy of the difference signal Δs is given as

$$e(\Delta t) = \sum_{i=1}^{N} \{\Delta s(t_i)\}^2 = \frac{N}{T}\int_0^T (\Delta t s'(t))^2 dt = \frac{N}{T} P \Delta t^2,$$

and as in the previous question, the expected value of the energy of the noise component parallel to the known waveform is $\frac{d^2}{2T} N$.

If we consider $E\left[\frac{N}{T} P \Delta t^2\right] = \frac{d^2}{2T} N$, we obtain

$$E[\Delta t^2] = \frac{d^2}{2P} \; .$$

A direct solution without sampling

Answer 2 of Problem 3

If the energy of signal $g(t)$ is defined as $\int_0^T \{g(t)\}^2 dt$, the energy of $\Delta s(t)$ is written as

$$e(\Delta t) = \int_0^T (\Delta t s'(t))^2 dt = P\Delta t^2.$$

On the other hand, the expected value of the energy of one frequency component (with a fixed phase) contained in the noise signal is

$$\frac{d^2}{2T} \cdot T = \frac{d^2}{2}.$$

If we consider the expected value of $e(\Delta t)$ is equal to this, that is,

$$E[P\Delta t^2] = \frac{d^2}{2}$$

we obtain the following equation:

$$E[\Delta t^2] = \frac{d^2}{2P}\ .$$

General expression

In a measurement system that estimates the parameter β from the measured value of the signal $s_\beta(n)\ \ (n = 1, 2, \ldots, N)$, we define the difference signal energy by $\Delta\beta$-shift of β as

$$e(\Delta\beta) = \sum_{n=1}^{N} \left\{ s_{\beta+\Delta\beta}(n) - s_\beta(n) \right\}^2 ,$$

while e_{W} denotes the expected value of the noise energy that is allocated to this error pattern $s_{\beta+\Delta\beta}(n) - s_\beta(n)$.
If $e(\Delta\beta)$ is larger than e_{W}, the change of $\Delta\beta$ can be detected.

Conversely, if $e(\Delta\beta)$ is smaller than e_{W}, it is impossible to discriminate from the data whether the observed change is due to the noise or due to a true shift of $\Delta\beta$.

Cramér–Rao bound

Consider estimating the unknown parameter θ from the observed value of the quantity x following a probability density function $f(x;\theta)$. Then, the variance of unbiased estimator $\hat{\theta}$ for θ is larger than the reciprocal of the Fisher information $I(\theta)$ as

$$V[\hat{\theta}] \geq \frac{1}{I(\theta)},$$

where $I(\theta) = E\left[\left(\frac{\partial}{\partial\theta}\ln f(x;\theta)\right)^2\right]$.

If we assume $f(x;\theta) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{(x-\theta)^2}{2\sigma^2}\right)$, we obtain

$\ln f(x;\theta) = -\frac{(x-\theta)^2}{2\sigma^2} - \ln\left(\sqrt{2\pi}\sigma\right)$, $\frac{\partial}{\partial\theta}\ln f(x;\theta) = \frac{x-\theta}{\sigma^2}$, $I(\theta) = \frac{1}{\sigma^2}$, and finally,

$$V[\hat{\theta}] \geq \sigma^2.$$

This implies that the variance of the estimator $\hat{\theta}$ is larger than σ^2 when a Gaussian noise of variance σ^2 is added to the true value θ.

If you can assume $\Delta s(n) = \Delta\boldsymbol{s} = \Delta\beta\boldsymbol{g}$ in the previous slide, the correspondence $(x,\theta) \to (\beta + \Delta\beta, \beta)$ and $\sigma^2 \to e_{\mathrm{W}}/||\boldsymbol{g}||^2$ leads to $V\left[\hat{\beta}\right] \geq e_{\mathrm{W}}/||\boldsymbol{g}||^2$.

What we have learned

When measuring a parameter under a certain noise, there is a measurement limit under that noise (and measurement time). If the accuracy of the measurement system in front of you have not reach the measurement limit, the measurement system can be improved by appropriate signal processing.
If the measurement limit has already been reached, you cannot improve the accuracy unless some new information about the nature of signal and noise is introduced.

A practical method to confirm amplitude measurement limit
* When the signal is a sine wave

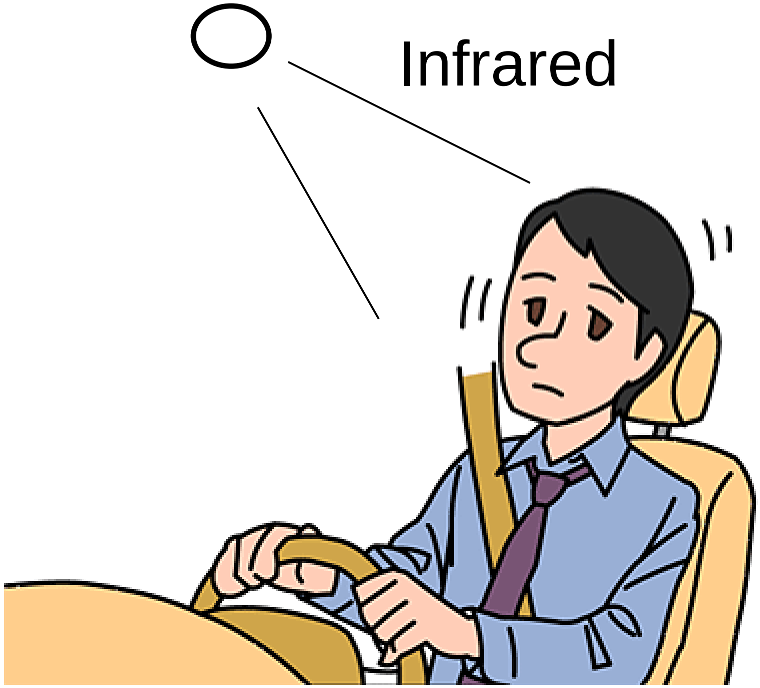
- Observe noise without signals, and let the measured pattern be $p(n)$ $(n = 1, \ldots, N)$.
- Obtain $e = \sum_{n=1}^{N} p(n)\phi(n)$ where $\phi(n) = \frac{2}{N}\cos(2\pi f \Delta t\, n)$ is the signal pattern and Δt is the sampling period. Note that $T = N\Delta t$.
- Repeat the above measurement and obtain the variation (standard deviation) of e, which provides the amplitude estimation error (standard deviation) of the signal. If higher accuracy is required, you must change the measurement system, increase the observation time, or find some new information on the models of signal and noise.

* We selected $\frac{2}{N}$ in $\phi(n) = \frac{2}{N}\cos(2\pi f \Delta t\, n)$ so that the sum of products with $p(n)$ provides the amplitude estimator of the sine wave.

* This method can be applied when $\phi(n)$ is not a sine wave.

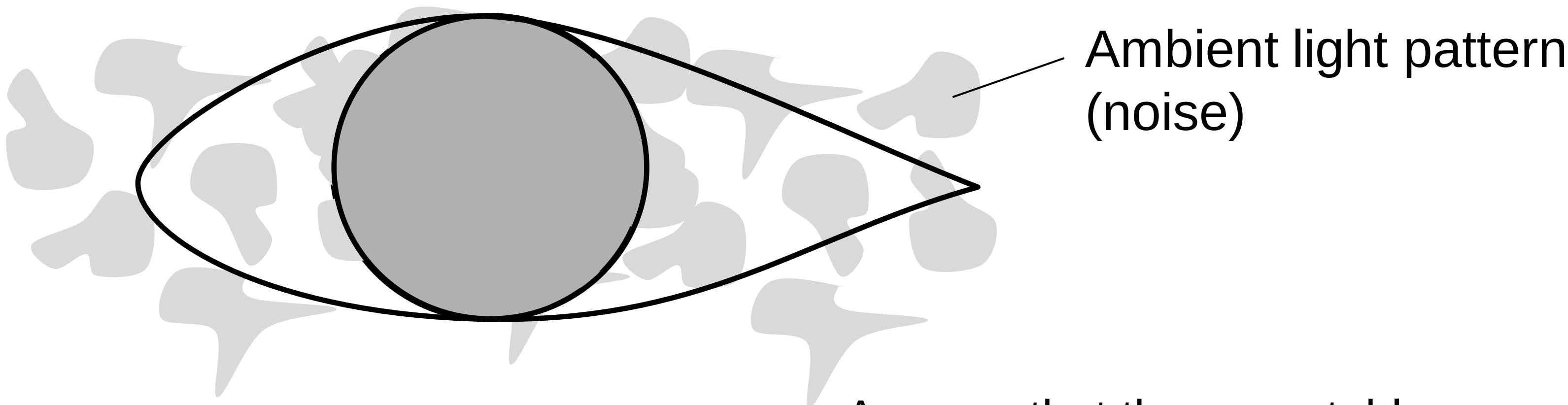
Example: Infrared measurement

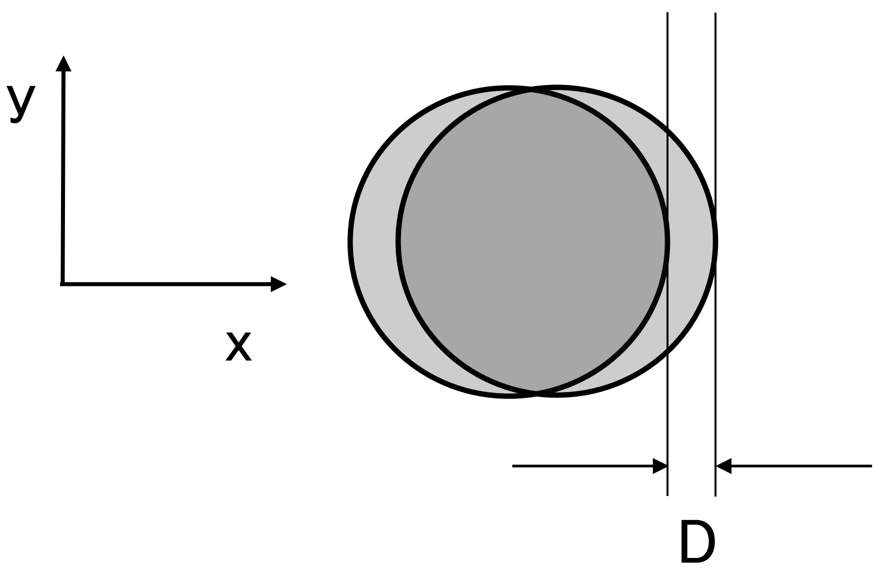
You are developing a system that measures eye positions from infrared images. You want to measure the eye position accurately every 1 ms, but the accuracy is insufficient under fluctuating ambient light.



Propose a method to suppress the influence on the measurement errors due to the ambient light.

Evaluation of measurement limits under random ambient light pattern





Assume that the acceptable measurement error is D in a certain application.

Consider the difference pattern

$$\Delta s = s(x - D, y) - s(x, y)$$

and its energy $e(D)$ regarding the image patterns of the eye $s(x, y)$.

If the energy $e(D)$ is larger than the noise energy e_{W} that is allocated to the same component (component parallel to Δs), the required accuracy can be attained just by improving the estimation algorithm.

Improving measurement accuracy

If the measurement limit has already been exceeded, you must improve the system physically or introduce new information about the signal/noise.
For example,

- Improve the infrared filter to heighten the light selectivity, to eliminate the ambient light with different wavelength from the used light.
- Modulate the infrared rays at a frequency that is not included in the ambient light and add a function to the camera to measure only the modulated frequency component.
- Introduce the regularity of human eye movement into the model.
- Find and utilize the hidden natures of ambient light.
- Use the components orthogonal to the noise pattern when the noise has a specific spatial pattern, or find the angle of the camera to weaken the influence of the ambient light pattern.

Question

Consider estimating the sound source direction from acoustic signals observed at two points.
Assume that it is a two-dimensional problem and the sound source is far enough.

The direction is estimated by the temporal difference of the signals observed at microphones 1 and 2.
Show the inevitable estimation error of the noise source direction assuming the following values.

- Observed signal at microphones 1 and 2:
  Sinusoidal wave of amplitude $1\ \mathrm{V}$ and $500\ \mathrm{Hz}$
- Observed signal duration: $0.1\ s$
- Noise at the microhphones: $0.01\ \ \mathrm{V}/\sqrt{\mathrm{Hz}}$